About the Chapter Project

Finding an average is something that most people can do almost instinctively. Currency exchange, hourly wage, car mileage, and average speed are common daily topics of discussion. You can measure an average in various ways. Two common averages are the arithmetic mean and the harmonic mean. In the Chapter Project, Means to an End, you will use the data provided to determine the most appropriate average.

After completing the Chapter Project, you will be able to do the following:

- Find the arithmetic mean and the harmonic mean of a set of data.
- Determine the relationship between the arithmetic and harmonic mean.
- Determine which of the averages—arithmetic mean, harmonic mean, or weighted harmonic mean—best represents a data set.

About the Portfolio Activities

Throughout the chapter, you will be given opportunities to complete Portfolio Activities that are designed to support your work on the Chapter Project.

- Exploring a historical representation of harmonic means is included in the Portfolio Activity on page 488.
- Exploring a geometric representation of harmonic means is included in the Portfolio Activity on page 497.
- Extending the definition of harmonic mean to \( n \) numbers is included in the Portfolio Activity on page 511.
- Using harmonic means to find average speeds is included in the Portfolio Activity on page 519.
Multiplying and Dividing Rational Expressions

Objectives
- Multiply and divide rational expressions.
- Simplify rational expressions, including complex fractions.

**APPLICATION**

**FUND-RAISING**

To analyze the revenue and costs from the sale of school-spirit ribbons, members of the Jamesville High School Home Economics Club used the revenue-to-cost ratio below.

\[
\frac{\text{revenue from the sale of each ribbon}}{\text{cost of making each ribbon}}
\]

They represented the number of ribbons produced and sold by \(x\) and the total production cost in dollars by \(0.8x + 25\). If the revenue for each ribbon was $3, for how many ribbons was the revenue-to-cost ratio 1.5 or greater? Finding the answer to this question involves writing and simplifying a rational expression. *You will answer this question in Example 6.*

**Simplifying Rational Expressions**

To *simplify* a rational expression, divide the numerator and the denominator by a common factor. The expression is simplified when you can no longer divide the numerator and denominator by a common factor other than 1.

**Example 1**

Simplify \(\frac{x^2 + 5x - 6}{x^2 - 36}\).

**Solution**

\[
\frac{x^2 + 5x - 6}{x^2 - 36} = \frac{(x + 6)(x - 1)}{(x - 6)(x + 6)}
\]

\[
= \frac{x - 1}{x - 6}
\]

Factor the numerator and denominator.

Divide out the common factor.

Note that 6 and \(-6\) are excluded values of \(x\) in the original expression.

**Try This**

Simplify \(\frac{b^2 - 49}{b^2 - 8b + 7}\).
Multiplying Rational Expressions

Multiplying rational expressions is similar to multiplying rational numbers.

<table>
<thead>
<tr>
<th>Rational Numbers</th>
<th>Rational Expressions</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{15}{4} \cdot \frac{14}{9} = \frac{3 \cdot 5 \cdot 2}{4 \cdot 7} = \frac{35}{6} )</td>
<td>( \frac{15}{x^2} \cdot \frac{4x^4}{21} = \frac{3 \cdot 5 \cdot 4 \cdot x^4}{3 \cdot 7 \cdot x^2} = \frac{20x^2}{7} )</td>
</tr>
</tbody>
</table>

**Example 2**

Simplify \( \frac{3}{4x^2} \cdot \frac{4x^3}{21} \cdot \frac{14}{4x^3} \).

**Solution**

\[
\frac{3}{4x^2} \cdot \frac{4x^3}{21} \cdot \frac{14}{4x^3} = \frac{3}{4} \cdot \frac{4}{3} \cdot \frac{2}{7} \cdot x^3 = \frac{1}{2x^3}
\]

**Try This**

Simplify \( \frac{28}{4a^3} \cdot \frac{4a^3}{21} \cdot \frac{3}{49a^3} \).

To multiply one rational expression by another, multiply as with fractions.

\[
\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}, \text{ where } b \neq 0 \text{ and } d \neq 0
\]

You can simplify the product by dividing out the common factors in the numerator and denominator before or after multiplying.

**Example 3**

Simplify \( \frac{x + 1}{x^2 + 2x - 3} \cdot \frac{x^2 + x - 6}{x^3 - 2x - 3} \).

**Solution**

\[
\frac{x + 1}{x^2 + 2x - 3} \cdot \frac{x^2 + x - 6}{x^3 - 2x - 3} = \frac{x + 1}{(x + 3)(x - 1)} \cdot \frac{(x + 3)(x - 2)}{(x - 3)(x + 1)}
\]

\[
= \frac{x - 2}{(x - 1)(x - 3)}, \text{ or } \frac{x - 2}{x^2 - 4x + 3}
\]

**Try This**

Simplify \( \frac{x^2 - 25}{x^2 - 5x + 6} \cdot \frac{x^2 - 4}{x^2 + 2x - 15} \).

**Dividing Rational Expressions**

Dividing one rational expression by another is similar to dividing one rational number by another.

<table>
<thead>
<tr>
<th>Rational Numbers</th>
<th>Rational Expressions</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{6}{8} + \frac{12}{32} = \frac{6}{8} + \frac{32}{12} )</td>
<td>( \frac{6}{x^2} + \frac{12}{x^3} = \frac{6}{x^2} + \frac{x^3}{12} )</td>
</tr>
</tbody>
</table>

Multiply by the reciprocal of \( \frac{12}{32} \).

\[
= \frac{6 \cdot 32}{8 \cdot 2} = \frac{4}{2}, \text{ or } 2
\]

Multiply by the reciprocal of \( \frac{12}{x^3} \).

\[
= \frac{6 \cdot x^3}{x^2 \cdot 12} = \frac{x^2}{2}, \text{ or } \frac{1}{2x^2}
\]
To divide one rational expression by another, multiply by the reciprocal of the divisor.

\[
\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}, \text{ where } b \neq 0, c \neq 0, \text{ and } d \neq 0
\]

Simplify by dividing out common factors in the numerator and denominator.

**Example 4** Simplify \(\frac{x-4}{(x-2)^2} + \frac{x^2-3x-4}{x^2-4}\).

**Solution**

\[
\frac{x-4}{(x-2)^2} + \frac{x^2-3x-4}{x^2-4} = \frac{x-4}{(x-2)^2} \cdot \frac{x^2-4}{x^2-3x-4}
\]

\[
= \frac{x-4}{(x-2)^2} \cdot \frac{(x-2)(x+2)}{(x-2)(x+2)}
\]

\[
= \frac{x+2}{(x-2)(x+1)}, \text{ or } \frac{x+2}{x^2-x-2}
\]

**Try This** Simplify \(\frac{(x+3)^2}{x-5} + \frac{x^2-9}{x^2-8x+15}\).

**Problem Solving**

You can use a graph to identify polynomials in a rational expression that cannot be factored with real numbers. For example, to determine whether \(x^2 - x + 1\) in the rational expression \(\frac{x-4}{x-1} \cdot \frac{x^2-x+1}{x^2}\) can be factored with real numbers, look for \(x\)-intercepts in the graph of \(y = x^2 - x + 1\).

The graph of \(y = x^2 - x + 1\) has no \(x\)-intercepts, so \(y = x^2 - x + 1\) has no real zeros and \(x^2 - x + 1\) cannot be factored with real numbers. Thus, the rational expression \(\frac{x-4}{x-1} \cdot \frac{x^2 - x + 1}{x^2}\) cannot be simplified further.

### Complex Fractions

A complex fraction is a quotient that contains one or more fractions in the numerator, the denominator, or both.

**Example 5** Simplify the complex fraction \(\frac{\frac{4a^2-1}{a^2-4}}{\frac{2a-1}{a+2}}\).

**Solution**

\[
\frac{\frac{4a^2-1}{a^2-4}}{\frac{2a-1}{a+2}} = \frac{4a^2-1}{a^2-4} \cdot \frac{a+2}{2a-1}
\]

\[
= \frac{(2a-1)(2a+1)}{(a-2)(a+2)} \cdot \frac{a+2}{2a-1}
\]

\[
= \frac{2a+1}{a-2}
\]

*Multiply by the reciprocal.*

*Factor.*

*Divide out common factors.*
TRY THIS

Simplify \( \frac{(x+2)^2}{x-3} \) \( \frac{x-3}{x^2-4} \) \( (x-3)^2 \).

CRITICAL THINKING

Use mental math to simplify \( \frac{x+y}{x-y} \). \( \frac{y+x}{y-x} \).

EXAMPLE 6

Refer to the revenue-to-cost ratio given at the beginning of the lesson.

For how many ribbons was the revenue-to-cost ratio 1.5 or greater?

SOLUTION

Revenue from the sale of each ribbon
\[ \frac{\text{cost of making each ribbon}}{0.8x + 25} \]

Simplify the complex fraction.
\[ \frac{3}{0.8x + 25} \cdot \frac{x}{0.8x + 25} = \frac{3x}{0.8x + 25} \]

Enter \( y = \frac{3x}{0.8x + 25} \) into a graphics calculator, and examine a table of values.

From the table, you can see that the revenue-to-cost ratio, \( y \), is greater than 1.5 when 21 or more ribbons, \( x \), were produced and sold.

Activity

Exploring Excluded Values in Quotients

You will need: a graphics calculator

1. Let \( f(x) = \frac{x-3}{x+2} \). When the complex fraction that defines \( f \) is simplified, it becomes \( x-3 \). Let \( g(x) = \frac{x-3}{x+2} \). Graph \( f \) and \( g \) on the same screen. What observations can you make about these graphs?

2. Make a table to evaluate \( f \) and \( g \) for \( x \)-values of \(-3, -2, -1, 0, 1, 2, \) and \( 3 \). How do the entries in the table compare?

3. Repeat Steps 1 and 2 for the functions \( f(x) = \frac{x^2-4}{x-1} \) and \( g(x) = \frac{x^2-9}{x-1} \).

4. Let \( f(x) = \frac{P}{Q} \) and \( g(x) = \frac{P}{R} \), where \( P, Q, \) and \( R \) are polynomials. Explain how to find the excluded values of \( f \) and why you should not try to find those values by examining \( g \).
Exercises

Communicate

1. In what ways is the multiplication of two rational expressions similar to the multiplication of two rational numbers?
2. In what ways is the division of two rational expressions similar to the division of two rational numbers?
3. Explain how to simplify a complex fraction such as \( \frac{x^2 - 1}{x^2 + 2x - 3} \). Compare the excluded values of \( x \) in the complex fraction and in its simplified form.

Guided Skills Practice

Simplify each rational expression. (EXAMPLES 1 AND 2)

4. \( \frac{x^2 - 25}{x^2 - 10x + 25} \)
5. \( \frac{4x^2}{5} \cdot \frac{30}{x^4} \cdot \frac{20x^3}{60} \)

Simplify each rational expression. (EXAMPLES 3 AND 4)

6. \( \frac{x^2 + 8x + 12}{x^2 + 2x - 15} \cdot \frac{x^2 + 8x + 15}{x^2 + 9x + 18} \)
7. \( \frac{x^2 - 2x + 1}{x^2 + 6x + 8} + \frac{x^2 - 1}{x^2 + 3x + 2} \)

8. Simplify the complex fraction \( \frac{2x - 6}{x^2 + 9x + 20} \). (EXAMPLES 5 AND 6)

Practice and Apply

Simplify each rational expression.

9. \( \frac{4x^2 + 8x + 4}{x + 1} \)
10. \( \frac{x^2 - 6x + 9}{x^2 - 9} \)
11. \( \frac{15}{x^2} \cdot \frac{x^5 - 4}{12} \cdot \frac{4}{x} \)
12. \( \frac{36x^2}{9x^2} \cdot \frac{12x^7}{2x} \cdot \frac{5}{x^2} \)
13. \( \frac{x^2 - 10x + 9}{x^2 + 2x - 3} \)
14. \( \frac{-x^2 - x + 6}{x^2 - 5x + 6} \)
15. \( \frac{x}{9x^8} \cdot \frac{x^7}{2x} \cdot \frac{45}{x^4} \)
16. \( \frac{-5}{x^3} \cdot \frac{-x^3}{3} \cdot \frac{-4}{x^3} \cdot \frac{20}{x} \)
17. \( \frac{x^2 - 4x - 5}{x^2 - 3x + 2} \cdot \frac{x^2 - 4}{x^2 - 3x - 10} \)
18. \( \frac{x^2 - 9}{x^2 - 4x + 4} \cdot \frac{x^2 - 4}{x^2 - x - 6} \)
19. \( \frac{2x^2 - 2x}{x^2 - 9} + \frac{x^2 + x - 2}{x^2 + 2x - 3} \)
20. \( \frac{4x^2 + 20x}{9 + 6x + x^2} + \frac{x + 5}{x^2 - 9} \)
21. \( \frac{x^4 + 2x^3 + x^2}{x^2 + x - 6} - \frac{x^2 - x - 2}{x^4 - x^2} \)
22. \( \frac{x^3 - 4x^3}{x^2 - x - 2} \cdot \frac{x^2 - 1}{x^3 - x^3 - 2x^3} \)
23. \( \frac{4x^3 - 9x}{2x - 7} + \frac{3x^3 + 2x^2}{4x^2 - 14x} \)
24. \( \frac{x^4 - 4x^2}{x^2 - 9} + \frac{4x^2 - 4x^3 + x^4}{x^2 - 6x + 9} \)
25. \( \frac{ax - bx + ay - by}{ax + bx + ay + by} \)
26. \( \frac{x^2 - y^2 - 4x + 4y}{x^2 - y^2 + 4x - 4y} \)
27. \( \frac{x^2}{4} \cdot \frac{(xy)^{-1}}{6} \cdot \frac{2y^2}{x} \)
28. \( 2rs + \frac{2r^2}{s} + \frac{2s^2}{r} \)
Simplify each expression.

29. \(\frac{(x+2)^2}{(x+3)^2} \div \frac{x+3}{x+2}\)  
30. \(\frac{x^2-4}{x^2-9} \div \frac{(x-2)^2}{(x-3)^2}\)  
31. \(\frac{x^2-9x+14}{x^2-6x+5} \div \frac{x^2-8x+7}{x^2-7x+10}\)

32. \(\frac{x^2+4x+3}{x^2+6x+8} \div \frac{x^2+9x+18}{x^2+7x+10}\)

33. \(\frac{x+2}{x+5} \div \frac{x+2}{x+1}\)  
34. \(\frac{1}{x^2} \div \frac{x}{x-7}\)

35. \(\frac{2x+3}{x-1} \div \frac{x}{3x-2}\)

36. \(\frac{x}{x^2-1} \div \frac{x+1}{x-1}\)

37. \(\frac{x+3}{x-1} \div \frac{1}{x(x-1)^{-1}}\)

38. \(\frac{2y+6}{y-7} \div \frac{3x}{2x+3}\)

39. \(\frac{(x+y)^2}{x+y} \div \frac{(x+y)^3}{x^2+2xy+y^2}\)

40. \(\frac{x+2y}{2x^2+3xy+y^2} \div \frac{2x^2+5xy+2y^2}{x+y}\)

41. \(\frac{1-7x^{-1}-18x^{-2}}{1-4x^{-2}}\)  
42. \(\frac{1+12x^{-1}+27x^{-2}}{x^{-1}+9x^{-2}}\)

43. \(\frac{(x+y)(x+y)^{-1}-2x(x+y)^{-1}}{(x-y)(x-y)^{-1}+2x(x-y)^{-1}}\)

44. Find the rational expression \(R\) whose numerator and denominator have degree 2 and leading coefficients of 1 such that \(\frac{x^2+3x-10}{x^2-8x+15} \cdot R = \frac{x-2}{x-3}\).

45. **GEOMETRY** An open-top box is to be made from a sheet of cardboard that is 20 inches by 16 inches. Squares with sides of \(x\) inches are to be cut on one side and creased on another to form tabs. When the sides are folded up, these tabs are glued to the adjacent sides to provide reinforcement.

   a. Show that \(x(20-2x)(16-2x)\) represents the volume of the box.

   b. Show that \(320-4x^2\) represents the surface area of the bottom and sides of the inside of the box.

   c. Write and simplify an expression for the ratio of the volume of the box to the inside surface area of the box.

   d. How does the ratio from part c change as \(x\) increases?

46. **ECONOMICS** It costs Emilio and Maria Vianco $1200 to operate their sandwich shop for one month. The average cost of preparing one sandwich is $1.69.

   a. Using the menu shown, find the average revenue per sandwich.

   b. Let \(x\) represent the number of sandwiches sold in one month. Write a function for the total cost, \(C\), of operating the business for one month by using the average cost of preparing one sandwich.

   c. Write a function for the ratio, \(R\), of average revenue per sandwich to the average cost per sandwich.
47. **Physics** The diagram below illustrates an ambulance traveling a definite distance in a specific period of time. The average acceleration, $a$, is defined as the change in velocity over the corresponding change in time.

![Diagram of an ambulance showing distance and time](image)

- **a.** Simplify the expression at right that defines $a$.
- **b.** If the distance, $d$, is measured in feet and the time, $t$, is measured in seconds, in what units is acceleration measured?

$$a = \frac{d_2 - d_1}{t_2 - t_1}$$

**Look Back**

Write an equation in slope-intercept form for the line that contains the given point and is perpendicular to the given line. *(Lesson 1.3)*

48. $(8, -4), y = -6x - 1$
49. $(3, 5), y = \frac{1}{5}x - 11$

Graph each piecewise function. *(Lesson 2.6)*

50. $f(x) = \begin{cases} x - 5 & \text{if } -1 < x \leq 4 \\ 9 - 2x & \text{if } 4 < x \leq 5 \end{cases}$
51. $g(x) = \begin{cases} -4 & \text{if } x < 0 \\ 2x - 4 & \text{if } 0 \leq x \leq 5 \\ -\frac{1}{3}x + 8 & \text{if } x > 5 \end{cases}$

Factor each expression. *(Lesson 5.3)*

52. $8x^2 - 4x$
53. $12x^2 - 3x + 6$
54. $12 - 4a - 22a^2$

Simplify each expression. Write your answer in the standard form for a complex number. *(Lesson 5.6)*

55. $\frac{2 + i}{3 + 2i}$
56. $\frac{4 - i}{6 - 3i}$
57. $\frac{3 - 2i}{5 + i}$

Write each product as a polynomial expression in standard form. *(Lesson 7.3)*

58. $x^2(x^2 - x^2 - 6x + 2)$
59. $(x - 2)(3x^3 - 6x - x^2)$
60. $(x^2 + 1)(2x^3 - 9)$

Factor each polynomial expression. *(Lesson 7.3)*

61. $x^3 - 1$
62. $125x^3 + 27$
63. $x^3 - 6x^2 - 8x$

**Look Beyond**

Simplify.

64. $\frac{3}{8} + \frac{1}{8}$
65. $\frac{3}{x} + \frac{1}{x}$
66. $\frac{3}{2x} + \frac{1}{x}$
67. $\frac{3}{2x} + \frac{1}{3x}$
A cab driver drove from the airport to a passenger’s home at an average speed of 55 miles per hour. He returned to the airport along the same highway at an average speed of 45 miles per hour. What was the cab driver’s average speed over the entire trip? The answer is not the average of 45 and 55. To answer this question, you need to add two rational expressions. You will answer this question in Example 5.

Adding two rational expressions with the same denominator is similar to adding two rational numbers with the same denominator.

**Rational Numbers**

\[
\frac{1}{7} + \frac{3}{7} = \frac{1 + 3}{7} = \frac{4}{7}
\]

**Rational Expressions**

\[
\frac{3}{x^2} + \frac{5}{x^2} = \frac{3 + 5}{x^2} = \frac{8}{x^2}
\]

**Example 1**

Simplify.

\(a. \ \frac{2x}{x + 3} + \frac{5}{x + 3}\)

\[
= \frac{2x + 5}{x + 3}
\]

\(b. \ -\frac{x^2}{x - 3} - \frac{9}{x - 3}\)

\[
= \frac{x^2 - 9}{x - 3}
\]

\[
= \frac{(x + 3)(x - 3)}{x - 3}
\]

\[
= x + 3
\]

Note that 3 is an excluded value of \(x\) in the original expression.

**Try This**

Simplify.

\(a. \ \frac{3x - 1}{2x - 1} + \frac{5 + 2x}{2x - 1}\)

\(b. \ -\frac{2x}{x - 5} - \frac{10}{x - 5}\)
To add two rational expressions with unlike denominators, you first need to find common denominators. The **least common denominator (LCD)** of two rational expressions is the **least common multiple (LCM)** of the denominators. The **least common multiple (LCM)** of two polynomials is the polynomial of lowest degree that is divisible by each polynomial.

Finding the LCM for two rational expressions is similar to finding the LCM for two rational numbers. Compare the procedures for rational numbers and for rational expressions shown below.

<table>
<thead>
<tr>
<th>Rational Numbers</th>
<th>Rational Expressions</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \frac{7}{300} + \frac{1}{90} = \frac{7}{300} \left( \frac{3}{3} \right) + \frac{1}{90} \left( \frac{10}{10} \right) ]</td>
<td>[ \frac{7}{3x^2} + \frac{1}{9x} = \frac{7}{3x^2} \left( \frac{3}{3} \right) + \frac{1}{9x} \left( \frac{x}{x} \right) ]</td>
</tr>
<tr>
<td>= [ \frac{21 + 10}{900} ]</td>
<td>= [ \frac{21 + x}{9x^2} ]</td>
</tr>
<tr>
<td>= [ \frac{31}{900} ]</td>
<td><strong>Least common denominator</strong></td>
</tr>
</tbody>
</table>

**Adding and Subtracting Rational Expressions**

To add or subtract two rational expressions, find a common denominator, rewrite each expression by using the common denominator, and then add or subtract. Simplify the resulting rational expression.

**EXAMPLE 2** Simplify \( \frac{x}{x - 2} + \frac{-8}{x^2 - 4} \).

**SOLUTION**

\[
\frac{x}{x - 2} + \frac{-8}{x^2 - 4} = \frac{x}{x - 2} + \frac{-8}{(x - 2)(x + 2)}
\]

The LCD is \((x - 2)(x + 2)\).

\[
= \frac{x(x + 2) - 8}{(x - 2)(x + 2)}
\]

Add the fractions.

\[
= \frac{x^2 + 2x - 8}{(x - 2)(x + 2)}
\]

Write the numerator in standard form.

\[
= \frac{(x + 4)(x - 2)}{(x - 2)(x + 2)}
\]

Factor the numerator.

\[
= \frac{(x + 4)(x - 2)}{(x - 2)(x + 2)}
\]

Divide out the common factors.

\[
= \frac{x + 4}{x + 2}
\]

Note that 2 and -2 are excluded values of \( x \) in the original expression.

**CHECKPOINT** Explain how factoring a polynomial can help you to add two rational expressions. Illustrate your response by simplifying \( \frac{x}{x - 3} + \frac{-50}{x^2 - 25} \).
Simplify \( \frac{6x}{3x - 1} - \frac{4x}{2x + 5} \).

**SOLUTION**

\[
\frac{6x}{3x - 1} - \frac{4x}{2x + 5} = \frac{6x(2x + 5) - 4x(3x - 1)}{(3x - 1)(2x + 5)}
\]

\[
= \frac{12x^2 + 30x - 12x^2 + 4x}{(3x - 1)(2x + 5)}
\]

\[
= \frac{34x}{(3x - 1)(2x + 5)}, \text{ or } \frac{34x}{6x^2 + 13x - 5}
\]

**CHECK**

Graph \( y = \frac{6x}{3x - 1} - \frac{4x}{2x + 5} \) and \( y = \frac{34x}{6x^2 + 13x - 5} \) together on the same screen to see if the graphs are the same.

You can also use a table of values to verify that the corresponding \( y \)-values are the same.

**CHECKPOINT ✔** Identify the excluded values of \( x \) for the original expression and for the simplified expression in Example 3. Are they the same or different? Explain.

**TRY THIS**

Simplify \( \frac{6}{x^2 - 2x} - \frac{1}{x^2 - 4} \).

Sometimes you need to rewrite complex fractions as rational expressions in order to add or subtract them, as shown in Example 4.

**EXAMPLE 4**

Simplify \( \frac{1}{1 + \frac{1}{a}} + \frac{1}{1 - \frac{1}{a}} \).

**SOLUTION**

\[
\frac{1}{1 + \frac{1}{a}} + \frac{1}{1 - \frac{1}{a}} = \frac{1}{a + 1} + \frac{1}{a - 1} = \frac{a}{a + 1} + \frac{a}{a - 1}
\]

\[
= \frac{a}{a + 1} + \frac{a}{a - 1}
\]

\[
= \frac{a}{a + 1} \left( \frac{a - 1}{a - 1} \right) + \frac{a}{a - 1} \left( \frac{a + 1}{a + 1} \right)
\]

\[
= \frac{a^2 - a + a^2 + a}{(a + 1)(a - 1)}
\]

\[
= \frac{2a^2}{a^2 - 1}
\]

**TRY THIS**

Simplify \( \frac{a}{a - \frac{1}{a}} - \frac{a}{a + \frac{1}{a}} \).
Refer to the cab driver’s round trip described at the beginning of the lesson. What is the cab driver’s average speed for the entire trip?

SOLUTION

Let \( d \) represent the length of the trip one way, let \( t_1 \) represent the cab driver’s travel time to the passenger’s home, and let \( t_2 \) represent his travel time back to the airport.

\[
\begin{align*}
\text{Home} & \quad \text{d} \quad \text{Airport} \\
\text{r}_1 & = 55 \text{ mph} & \quad \text{d} = r_1 t_1 \\
\text{d} & = r_2 t_2 & \quad \text{t}_1 = \frac{d}{r_1} = \frac{d}{55} \quad \text{and} \quad \text{t}_2 = \frac{d}{r_2} = \frac{d}{45}
\end{align*}
\]

average speed = \( \frac{\text{total distance}}{\text{total time}} \)

\[
= \frac{d + d}{t_1 + t_2}
= \frac{d}{\frac{d}{55} + \frac{d}{45}}
= \frac{2d}{\frac{9d}{495}}
= \frac{2d \times 495}{20d}
= 49.5
\]

Thus, the cab driver’s average speed was 49.5 miles per hour. The average speed was less than the average of 45 and 55 because he spent more time driving at 45 miles per hour than at 55 miles per hour.

CRITICAL THINKING

Suppose that the cab driver travels to the passenger’s home at \( a \) miles per hour and returns along the same route at \( b \) miles per hour. Show that his average speed for the entire trip is not simply \( \frac{a + b}{2} \).

Exercises

Communicate

1. Explain how to find the least common denominator in order to add \( \frac{x + 5}{x^2 - 7x + 6} + \frac{x - 1}{x^2 - 36} \).

2. Explain how to use a graph to check your answer when you add two rational expressions.

3. Choose the two expressions below that are equivalent and explain why they are equivalent.

   a. \( \frac{3 + 7}{x^2 + 4} \)   b. \( \frac{10}{x^2} + \frac{10}{4} \)   c. \( \frac{3}{x^2} + \frac{7}{4} \)   d. \( \frac{3}{x^2 + 4} + \frac{7}{x^2 + 4} \)
Guided Skills Practice

Simplify. (EXAMPLES 1 AND 2)

4. \( \frac{3x}{x - 1} + \frac{2}{x - 1} \)

5. \( \frac{3x + 5}{x + 2} - \frac{x + 1}{x + 2} \)

6. \( \frac{12}{x^2 - 1} + \frac{4}{x + 1} \)

Simplify. (EXAMPLES 3 AND 4)

7. \( \frac{x + 1}{2x - 1} - \frac{2x + 1}{x - 1} \)

8. \( \frac{1}{1 - \frac{1}{7}} \)

9. **TRAVEL** Refer to the cab driver’s round trip described at the beginning of the lesson. What is the average speed for the entire trip if he drives to the passenger’s home at 52 miles per hour and returns to the airport along the same route at 38 miles per hour? (EXAMPLE 5)

Practice and Apply

Simplify.

10. \( \frac{2x - 3}{x + 1} + \frac{6x + 5}{x + 1} \)

11. \( \frac{7x - 13}{2x - 1} + \frac{x + 9}{2x - 1} \)

12. \( \frac{r + 9}{4} + \frac{r - 3}{2} \)

13. \( \frac{x + 7}{3} - \frac{4x + 1}{9} \)

14. \( \frac{x}{x^2 - 4} - \frac{2}{x - 2} \)

15. \( \frac{2x}{x + 3} - \frac{x - 3}{x^2 + 6x + 9} \)

16. \( \frac{4}{x^5} + \frac{5}{x + 3} \)

17. \( \frac{2}{x + 2} - \frac{6}{x - 2} \)

18. \( \frac{3}{x - 1} - \frac{2}{x + 1} \)

19. \( \frac{8}{3x - 5} + \frac{7}{2x + 3} \)

20. \( \frac{2x + 3}{x + 3} + \frac{x}{x - 2} \)

21. \( \frac{x + 2}{2x} - \frac{2x}{x - 1} \)

22. \( x^2 + \frac{2x}{3x - 5} \)

23. \( \frac{x + 1}{(x - 1)^2} + \frac{x - 2}{x - 1} \)

24. \( 2x^2 - 1 - \frac{x - 1}{x + 2} \)

25. \( \frac{3}{2x - 1} \)

26. \( \frac{3x + 1}{2} \)

27. \( \frac{x - 1}{2} + \frac{3}{x - 1} \)

28. \( \frac{4}{x + 2} - \frac{3}{x + 2} \)

29. \( \frac{x + 2}{x + 5} \)

30. \( \frac{2x + 10}{x + 5} - \frac{4}{x + 1} \)

31. \( \frac{1 - xy^{-1}}{x^{-1} - y^{-1}} \)

32. \( \frac{x - y}{x^{-1} - y^{-1}} \)

33. \( \frac{1}{a^2} - \frac{1}{b^2} \)

Write each expression as a single rational expression in simplest form.

34. \( \frac{3x}{x - 1} + \frac{5x + 2}{x - 1} - \frac{10}{x - 1} \)

35. \( \frac{7x}{x^2 - 1} - \frac{x}{x^2 - 1} + \frac{6}{x^2 - 1} \)

36. \( \frac{7}{x + 7} + \frac{-x}{x - 7} + \frac{2x}{x^2 - 49} \)

37. \( \frac{x}{x - 3} - \frac{3}{x + 4} + \frac{7}{x^2 + x - 12} \)

38. \( (a - b)^{-1} - (a + b)^{-1} \)

39. \( (a - b)^{-2} - (a + b)^{-2} \)

40. \( \frac{x}{x - y} - \frac{x^2 + y^2}{x^2 - y^2} + \frac{y}{x + y} \)

41. \( \frac{3r}{2r - s} - \frac{2r}{2r + s} + \frac{2x^2}{4r^2 - s^2} \)

Find numbers \( A, B, C, \) and \( D \) such that the given rational expression equals the sum of the two simpler rational expressions, as indicated.

42. \( \frac{3x + 2}{x - 5} = \frac{Ax}{x - 5} + \frac{D}{x - 5} \)

43. \( \frac{-x + 1}{(x - 2)(x - 3)} = \frac{B}{x - 2} + \frac{D}{x - 3} \)

44. \( \frac{2x^2 + 5}{x^2 + 11x + 30} = \frac{Ax}{x + 5} + \frac{Cx + D}{x + 6} \)

45. \( \frac{x^2 - 7}{x^2 + 2x - 3} = \frac{Ax}{x + 3} + \frac{Cx + D}{x - 1} \)
46. **GEOMETRY** In the diagram at right, square $A$ is 1 unit on a side, square $B$ is $\frac{1}{2}$ of a unit on a side, square $C$ is $\frac{1}{4}$ of a unit on a side, and so on.

a. Write a sum for the total area of squares $A$, $B$, $C$, and $D$, using only powers of 2.

b. Rewrite the sum you wrote in part a as a single rational number.

c. Suppose that two more squares, $E$ and $F$, are added to the set of squares, continuing the pattern. Write a single rational number for the total area of squares $A$ through $F$.

d. Convert your answers from parts b and c to decimals rounded to the nearest ten-thousandth. What common fraction do the answers appear to be getting closer and closer to?

47. **ELECTRICITY** The effective resistance, $R_T$, of parallel resistors in an electric circuit equals the reciprocal of the sum of the reciprocals of the individual resistances.

$$R_T = \frac{1}{\frac{1}{R_A} + \frac{1}{R_B} + \frac{1}{R_C}}$$

Resistance in an electric circuit is measured in ohms.

a. A circuit has three parallel resistors, $R_A$, $R_B$, and $R_C$. Find $R_T$ to the nearest hundredth, given $R_A = 5$ ohms, $R_B = 8$ ohms, and $R_C = 12$ ohms.

b. Write $R_T$ as a rational expression with no fractions in the denominator.

48. Justine’s average speed for a trip from $A$ to $C$ and back to $A$

49. Justine’s average speed for a trip from $B$ to $D$ and back to $B$

50. Justine’s average speed for a trip from $A$ to $D$ and back to $A$
State the property that is illustrated in each statement. All variables represent real numbers. (LESSON 2.1)

51. $-8x(5x + 2) = -40x^2 - 16x$  
52. $(3 - x)12x = 12x(3 - x)$

Find the discriminant, and determine the number of real solutions. Then solve. (LESSON 5.6)

53. $0 = x^2 - 3x + 4$  
54. $x^2 - 2x + 1 = 0$  
55. $-2x^2 - 5x + 12 = 0$

56. PHYSICAL SCIENCE  When sunlight strikes the surface of the ocean, the intensity of the light beneath the surface decreases exponentially with the depth of the water. If the intensity of the light is reduced by 75% for each meter of depth, what expression represents the intensity of light beneath the surface? (LESSON 6.2)

Write each expression as a single logarithm. Then evaluate. (LESSON 6.4)

57. $\log_2 32 - \log_2 8$  
58. $\log_2 4^3 + \log_2 16$

Use a graph and the Location Principle to find the real zeros of each function. (LESSON 7.4)

59. $d(x) = x^3 - 6x^2 + 5x + 12$  
60. $f(x) = x^3 - 2x^2 - 11x + 12$

61. $f(x) = x^3 + 2x^2 - 5x - 6$  
62. $g(x) = x^3 + 8x^2 + 4x - 48$

Find all real solutions of the rational equation $1.4 = \frac{(x + 3)(x - 1)}{x^2 - 1}$. Be sure to check for excluded values. (LESSON 8.4)

PORTFOLIO ACTIVITY

The definition of a harmonic mean may be extended for 3, 4, or $n$ numbers. For any three numbers $a$, $b$, and $c$, the harmonic mean is $\frac{3}{\frac{1}{a} + \frac{1}{b} + \frac{1}{c}}$.

1. Simplify this complex fraction.
2. Find the harmonic mean of the numbers 3, 4, and 5.

For any four numbers $a$, $b$, $c$, and $d$, the harmonic mean is $\frac{4}{\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d}}$.

3. Simplify this complex fraction.
4. Find the harmonic mean of the numbers 2, 4, 6, and 8.

WORKING ON THE CHAPTER PROJECT

You should now be able to complete Activity 2 of the Chapter Project.
Objectives

- Solve a rational equation or inequality by using algebra, a table, or a graph.
- Solve problems by using a rational equation or inequality.

Why There are many events in the real world that can be represented by a rational equation or inequality. For example, you can write a rational equation to represent speed and distance information for a triathlon.

APPLICATION SPORTS

Rachel finished a triathlon involving swimming, bicycling, and running in 2.5 hours. Rachel’s bicycling speed was about 6 times her swimming speed, and her running speed was about 5 miles per hour greater than her swimming speed. To find the speeds at which Rachel competed, you can solve a rational equation. A rational equation is an equation that contains at least one rational expression.

<table>
<thead>
<tr>
<th>Distance (mi)</th>
<th>Speed (mph)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Swimming</td>
<td>$d_s = 0.5$</td>
</tr>
<tr>
<td>Bicycling</td>
<td>$d_b = 25$</td>
</tr>
<tr>
<td>Running</td>
<td>$d_r = 6$</td>
</tr>
</tbody>
</table>

**EXAMPLE 1** Find the speeds at which Rachel competed if she finished the triathlon in 2.5 hours.

**SOLUTION**

1. Find the time for each part of the triathlon.

   **Swimming time**
   
   $d_s = rt_s$

   $0.5 = st_s$

   $\frac{0.5}{s} = t_s$

   **Bicycling time**

   $d_b = rt_b$

   $25 = (6s)t_b$

   $\frac{25}{6s} = t_b$

   **Running time**

   $d_r = rt_r$

   $6 = (s + 5)t_r$

   $\frac{6}{s + 5} = t_r$
2. Write a rational function to represent the total time, \( T \), in hours for the triathlon in terms of the swimming speed, \( s \), in miles per hour.

\[
T(s) = t_s + t_b + t_r
\]

\[
T(s) = \frac{0.5}{s} + \frac{25}{6s} + \frac{6}{s + 5}
\]

\[
T(s) = \frac{0.5}{s} \left[ \frac{6(s + 5)}{6s} + \frac{25}{s + 5} + \frac{6}{s + 5} \right]
\]

\[
T(s) = \frac{3s + 15 + 25s + 125 + 36s}{6s(s + 5)}
\]

The LCD is \( 6s(s + 5) \).

3. Solve the rational equation \( 2.5 = \frac{64s + 140}{6s(s + 5)} \).

Graph \( y = \frac{64x + 140}{6x(x + 5)} \) and \( y = 2.5 \), and find the \( x \)-coordinate of the point of intersection.

Thus, Rachel swam at about 2.7 miles per hour, bicycled at about 6 \( \cdot \) 2.7, or 16.2, miles per hour, and ran at about 5 + 2.7, or 7.7, miles per hour.

**CRITICAL THINKING**

How can you solve \( 2.5 = \frac{64x + 140}{6x(x + 5)} \) by using the quadratic formula?

**Example 2**

Solve \( \frac{x}{x - 6} = \frac{1}{x - 4} \).

**Solution**

**Method 1** Use algebra.

\[
\frac{x}{x - 6} = \frac{1}{x - 4}, \quad x \neq 6, x \neq 4
\]

\[
x(x - 4) = 1(x - 6)
\]

\[
x^2 - 4x = x - 6
\]

\[
x^2 - 5x + 6 = 0
\]

\[
(x - 2)(x - 3) = 0
\]

\[
x = 2 \quad \text{or} \quad x = 3
\]

**Check**

Let \( x = 2 \).

\[
\frac{x}{x - 6} = \frac{1}{x - 4}
\]

\[
\frac{2}{2 - 6} = \frac{1}{2 - 4}
\]

\[
-\frac{1}{2} = -\frac{1}{2} \quad \text{True}
\]

Let \( x = 3 \).

\[
\frac{x}{x - 6} = \frac{1}{x - 4}
\]

\[
\frac{3}{3 - 6} = \frac{1}{3 - 4}
\]

\[
-1 = -1 \quad \text{True}
\]

The solutions are 2 and 3.

**Method 2** Use a graph.

Because it is not easy to see the intersection of \( y = \frac{x}{x - 6} \) and \( y = \frac{1}{x - 4} \), use another graphing method.

Write \( \frac{x}{x - 6} = \frac{1}{x - 4} \) as \( \frac{x}{x - 6} - \frac{1}{x - 4} = 0 \).

Then graph \( y = \frac{x}{x - 6} - \frac{1}{x - 4} \), and find the zeros of the function.
Sometimes solving a rational equation introduces extraneous solutions. An extraneous solution is a solution to a resulting equation that is not a solution to the original equation. Therefore, it is important to check your answers, as shown in Example 3.

**Example 3** Solve \( \frac{x}{x-3} + \frac{2x}{x+3} = \frac{18}{x^2 - 9} \).

**Solution**

**Method 1** Use algebra.

Multiply each side of the equation by the LCD, \((x - 3)(x + 3)\), or \(x^2 - 9\).

\[
\frac{x}{x-3} + \frac{2x}{x+3} = \frac{18}{x^2 - 9}, \quad \text{where } x \neq 3 \text{ and } x \neq -3
\]

\[
\frac{x}{x-3}(x+3)(x-3) + \frac{2x}{x+3}(x+3)(x-3) = \frac{18}{x^2 - 9}(x+3)(x-3)
\]

\[
x(x+3) + 2x(x-3) = 18
\]

\[
x^2 + 3x + 2x^2 - 6x = 18
\]

\[
3x^2 - 3x - 18 = 0
\]

\[
3(x^2 - x - 6) = 0
\]

\[
3(x - 3)(x + 2) = 0
\]

\[
x = 3 \quad \text{or} \quad x = -2
\]

**Check**

Since \( x = 3 \) is an excluded value of \( x \) in the original equation, it is an extraneous solution. Check \( x = -2 \).

\[
\frac{x}{x-3} + \frac{2x}{x+3} = \frac{18}{x^2 - 9}
\]

\[
\frac{-2}{-2 - 3} + \frac{2(-2)}{-2} + 3 \frac{3}{(-2)^2 - 9}
\]

\[
-3 \frac{3}{5} = \frac{18}{5} \quad \text{True}
\]

Thus, the only solution is \( x = -2 \).

**Method 2** Use a graph.

Because it is not easy to see the intersection of \( y = \frac{x}{x-3} + \frac{2x}{x+3} \) and \( y = \frac{18}{x^2 - 9} \), use another graphing method.

Write \( \frac{x}{x-3} + \frac{2x}{x+3} = \frac{18}{x^2 - 9} \) as \( \frac{x}{x-3} + \frac{2x}{x+3} - \frac{18}{x^2 - 9} = 0 \). Then graph \( y = \frac{x}{x-3} + \frac{2x}{x+3} - \frac{18}{x^2 - 9} \), and find any zeros of the function.

The solution is \( x = -2 \).

**Try This**

Solve \( \frac{x}{x-2} + \frac{x}{x-3} = \frac{3}{x^2 - 5x + 6} \).

**Critical Thinking**

Explain why an extraneous solution is obtained in Example 3 above.
A **rational inequality** is an inequality that contains at least one rational expression.

### Activity

**Solving Rational Inequalities**

**You will need:** a graphics calculator

1. Graph \( y_1 = \frac{x + 2}{x - 4} \).
2. Use the table feature and the graph to identify the values of \( x \) for which \( y_1 \) is 0, \( y_1 \) is undefined, \( y_1 \) is positive, and \( y_1 \) is negative.
3. On the same screen, graph \( y_2 = 2x - 11 \).
4. For what values of \( x \) is \( y_1 = y_2 \)? \( y_1 < y_2 \)? \( y_1 > y_2 \)?
5. Explain how to use a graph and a table of values to solve \( \frac{x + 2}{x - 4} < 2x - 11 \) and \( \frac{x + 2}{x - 4} > 2x - 11 \).

### Example 4

**Solve** \( \frac{x}{2x - 1} \leq 1 \).

**Solution**

**Method 1** Use algebra.

To clear the inequality of fractions, multiply each side by \( 2x - 1 \). You must consider both cases: \( 2x - 1 \) is positive or \( 2x - 1 \) is negative.

\[
\begin{align*}
\frac{x}{2x - 1} & \leq 1, \text{ where } 2x - 1 > 0 \\
\frac{x}{2x - 1} & \leq 1, \text{ where } 2x - 1 < 0
\end{align*}
\]

\[
\begin{align*}
x & \leq 2x - 1 \\
x & \geq 2x - 1 \\
x & \geq 1 \\
x & \leq 1 \\
\text{Change } \leq \text{ to } \geq.
\end{align*}
\]

For this case, \( x > \frac{1}{2} \) because \( 2x - 1 > 0 \). Therefore, the solution must satisfy \( x \geq 1 \) and \( x > \frac{1}{2} \).

For this case, \( x \geq 1 \).

Thus, the solution is \( x \geq 1 \text{ or } x < \frac{1}{2} \).

**Method 2** Use a graph.

Graph \( y = \frac{x}{2x - 1} \) and \( y = 1 \), and find the values of \( x \) for which the graph of \( y = \frac{x}{2x - 1} \) is below the graph of \( y = 1 \).

Thus, the solution is \( x \geq 1 \text{ or } x < \frac{1}{2} \).

**TRY THIS**

Solve \( \frac{x - 1}{x + 2} < 3 \).
Solve \( \frac{x - 2}{2(x - 3)} > \frac{x}{x + 3} \).

**SOLUTION**

In order to clear the inequality of fractions, you can multiply each side by the LCD, \( 2(x - 3)(x + 3) \). You must consider four possible cases:

- **Case 1:** \( x - 3 \) is positive and \( x + 3 \) is positive,
- **Case 2:** \( x - 3 \) is positive and \( x + 3 \) is negative,
- **Case 3:** \( x - 3 \) is negative and \( x + 3 \) is positive,
- **Case 4:** \( x - 3 \) is negative and \( x + 3 \) is negative.

The algebraic method of solution is beyond the scope of this textbook, but with a graphics calculator, the solution is much easier to find.

Rewrite \( \frac{x - 2}{2(x - 3)} > \frac{x}{x + 3} \) as \( \frac{x - 2}{2(x - 3)} - \frac{x}{x + 3} > 0 \).

Graph \( y = \frac{x - 2}{2(x - 3)} - \frac{x}{x + 3} \) and find the values of \( x \) for which \( y > 0 \).

The graph shows that there are two intervals of \( x \) for which \( y > 0 \). These two intervals can be found by using a table of values.

One interval for which \( y > 0 \) is \(-3 < x < 1\), as shown above. The other interval for which \( y > 0 \) is \( 3 < x < 6 \), as shown below.

Thus, the solution is \(-3 < x < 1 \) or \( 3 < x < 6 \).

**TRY THIS**

Solve \( \frac{x + 1}{x - 1} < -\frac{x}{x - 1} \).
Exercises

Communicate

1. Explain what an extraneous solution is and how you can tell whether a solution to a rational equation is extraneous.

2. Explain how to use a graph to check the solutions to a rational equation that are obtained by using algebra.

3. Explain how to use the graphs of \( y = \frac{x-1}{x+2} \)
and \( y = 3 \), shown at right, to solve \( \frac{x-1}{x+2} < 3 \)
and \( \frac{x-1}{x+2} > 3 \).

Guided Skills Practice

APPLICATION

4 | SPORTS  Refer to Rachel’s triathlon information given at the beginning of
the lesson. At what swimming, bicycling, and running speeds must Rachel
compete in order to finish the triathlon in 2 hours? (EXAMPLE 1)

Solve each equation.

5. \( \frac{2x-1}{x} = \frac{3}{x+2} \) (EXAMPLE 2)

6. \( \frac{2}{x-1} + \frac{2}{x+1} = \frac{-4}{x^2-1} \) (EXAMPLE 3)

Solve each inequality.

7. \( \frac{2x-3}{x} \geq 2 \) (EXAMPLE 4)

8. \( \frac{1}{x+2} < \frac{-1}{x+3} \) (EXAMPLE 5)

Practice and Apply

Solve each equation. Check your solution.

9. \( \frac{x+3}{2x} = \frac{5}{8} \)

10. \( \frac{2y-1}{4y} = \frac{4}{6} \)

11. \( \frac{4}{n+4} = 1 \)

12. \( \frac{-6}{m-3} = 1 \)

13. \( \frac{1}{3z} + \frac{1}{8} = \frac{4}{3z} \)

14. \( \frac{1}{t} + \frac{1}{3} = \frac{8}{3t} \)

15. \( \frac{y+3}{y-1} = \frac{y+2}{y-3} \)

16. \( \frac{2n+1}{3n+4} = \frac{2n-8}{3n+8} \)

17. \( \frac{x+3}{x} + 1 = \frac{x+5}{x} \)

18. \( \frac{2x-1}{x+3} = \frac{x}{x+3} \)

19. \( \frac{x+1}{x-1} + \frac{2}{x} = \frac{x}{x+1} + \frac{2}{x} = \frac{x}{x+1} \)

20. \( \frac{3}{x+2} - \frac{x}{1} = \frac{4}{3} \)

21. \( \frac{1}{6} - \frac{1}{x} = \frac{4}{3x^2} \)

22. \( \frac{1}{1+c} - \frac{1}{2+c} = \frac{1}{4} \)

23. \( \frac{2x+3}{x-1} - \frac{2x-3}{x+1} = \frac{10}{x^2-1} \)

24. \( \frac{x-4}{x+2} + \frac{2}{x-2} = \frac{17}{x^2-4} \)

25. \( \frac{b}{b+3} - \frac{b}{b-2} = \frac{10}{b^2+b-6} \)

26. \( \frac{3z}{z-1} + \frac{2z}{z-6} = \frac{5z^2-15z+20}{z^2-7z+6} \)

27. \( \frac{3}{x+2} + \frac{12}{x^2-4} = \frac{-1}{x-2} \)

28. \( \frac{x+2}{2x-3} - \frac{x-2}{2x+3} = \frac{21}{4x^2-9} \)
Solve each inequality. Check your solution.

29. \( \frac{x + 3}{3x} > 2 \)
30. \( \frac{x + 5}{4x} > 3 \)
31. \( \frac{x - 5}{3x} < -3 \)
32. \( \frac{x - 5}{3x} < 3 \)
33. \( \frac{2x + 1}{x - 2} > 4 \)
34. \( 3 < \frac{3x + 4}{2 + 1} \)
35. \( \frac{x + 1}{x} \leq \frac{1}{2} \)
36. \( -\frac{1}{2} \geq \frac{1}{x - 4} \)
37. \( \frac{7x}{3x + 2} < 2 \)

Use a graphics calculator to solve each rational inequality. Round answers to the nearest tenth.

38. \( \frac{1}{2} > x^2 \)
39. \( \frac{1}{x} \leq x^2 - 1 \)
40. \( \frac{x - 2}{x - 1} \geq 2x \)
41. \( x^2 - 4 \leq \frac{1}{x^2} \)
42. \( 2x + 1 \geq \frac{1}{2x + 1} \)
43. \( \frac{t}{t - 1} - \frac{2}{t + 1} \leq \frac{5}{t^2 - 1} \)
44. \( x + \frac{1}{x - 1} + \frac{2}{x} \geq 1 \)
45. \( \frac{a - 3}{3a} \geq \frac{1}{3a^2 + 9a} + \frac{1}{a + 3} \)
46. \( \frac{x^2 + 1}{(x - 1)^2} > \frac{1}{x} \)

State whether each equation is always true, sometimes true, or never true.

47. \( \frac{2x + 8}{x^2 - 16} = -\frac{2}{x - 4}, x \neq \pm 4 \)
48. \( 1 - 5x^{-1} + 4x^{-2} \leq \frac{x - 1}{x + 4}, x \neq 0 \) or \( \pm 4 \)
49. \( \frac{3}{x + 2} + \frac{12}{x^2 - 4} = -\frac{1}{x - 2} \)
50. \( 2x + 3 \cdot \frac{2x - 3}{x + 1} = \frac{10}{x^2 - 1} \)
51. \( \frac{x}{x + 4} > 2x + 6 \)
52. \( \frac{x - 6}{x^2 - 2x - 8} + \frac{3}{x - 4} \leq \frac{2}{x + 2} \)

**Challenge**

53. Solve \( \frac{3}{(x - 1)^2} > 0 \) by using mental math.

54. **Cultural Connection: Asia** A ninth-century Indian mathematician, Mahavira, posed the following problem:

   *There are four pipes leading into a well. Individually, four pipes can fill the well in \( \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \text{ and } \frac{1}{5} \) of a day. How long would it take for the pipes to fill the well if they were all working simultaneously, and what fraction of the well would be filled by each pipe?*

**Connection**

55. **Geometry** The length of a rectangle is 5 more than its width. Find the length and the width of the rectangle if the ratio of the length to the width is at least 1.5 and no more than 3.

**Applications**

56. **Sports** Michael is training for a triathlon. He swims 0.6 miles, bicycles 15 miles, and runs 8 miles. Michael bicycles about 9 times as fast as he swims, and he runs about 6 miles per hour faster than he swims.

   a. Write a rational function, in terms of swimming speed, for the total time it takes Michael to complete his workout.

   b. Find the speeds at which Michael must swim, run, and bike to complete his workout in 1.5 hours.

57. **Physics** An object weighing \( w_0 \) kilograms on Earth is \( h \) kilometers above Earth. The function that represents the object’s weight at that altitude is \( w(h) = w_0 \left( \frac{6400}{6400 + h} \right)^2 \). Find the approximate altitude of a satellite that weighs 3500 kilograms on Earth and 1200 kilograms in space.
Look Back

Evaluate each expression. (LESSON 2.2)

58. \( \frac{81}{2} \)  
59. \( 13^0 \)  
60. \( 9^{\frac{3}{2}} \)  
61. \( 27^{\frac{1}{3}} \)

Find the inverse of each function. State whether the inverse is a function. (LESSON 2.5)

62. \( \{ (3, 5), (2, 8), (1, 5), (0, 3) \} \)
63. \( \{ (-1, -4), (-2, -3), (-3, -2), (0, -1) \} \)
64. \( g(x) = \frac{1}{4}x - 5 \)
65. \( h(x) = \frac{5 - x}{2} \)

Identify each transformation from the parent function \( f(x) = x^2 \) to \( g \). (LESSON 2.7)

66. \( g(x) = -2x^2 \)
67. \( g(x) = (x - 2)^2 \)
68. \( g(x) = \frac{1}{2}(x + 3)^2 \)
69. \( g(x) = 3x^2 - 5 \)
70. \( g(x) = (-2x)^2 + 1 \)
71. \( g(x) = 2(4 - x)^2 - 6 \)

Look Beyond

[72] Graph the functions \( f(x) = \sqrt{x} \), \( g(x) = \sqrt[3]{x} \), \( h(x) = \sqrt[4]{x} \), and \( k(x) = \sqrt[5]{x} \).

How are they alike? How are they different? (Hint: Use the fact that \( \sqrt[n]{x} = x^{\frac{1}{n}} \).)

Refer to Example 5 on page 508. Notice that the cab driver’s average speed is the total distance divided by the total time. The harmonic mean of the two speeds, 45 miles per hour and 55 miles per hour, gives the average speed for the entire trip.

1. Find the harmonic mean of 45 and 55.

2. How does your answer to Step 1 compare with the average speed found in Example 5?

Justin cycles for \( \frac{4}{2} \) hours. He cycles along a level road for 24 miles, and then he cycles up an incline for 24 miles more. Justin immediately turns around and cycles back to his starting point along the same route. Justin cycles on level ground at a rate of 24 miles per hour, uphill at a rate of 12 miles per hour, and downhill at a rate of 48 miles per hour.

3. Explain why Justin’s average speed over the entire trip is the harmonic mean of 24, 12, 48, and 24.

4. Find Justin’s average speed over the entire trip.

WORKING ON THE CHAPTER PROJECT

You should now be able to complete the Chapter Project.
LESSON 8.2

Key Skills

Identify all excluded values, asymptotes, and holes in the graph of a rational function.

\[ y = \frac{4x^2 + 12x}{x^2 + x - 6} = \frac{4x(x + 3)}{(x - 2)(x + 3)} \]

The excluded values are \( x = 2 \) and \( x = -3 \).

\( x + 3 \) is a factor of the numerator and the denominator, so the graph has a hole when \( x = -3 \).

\( x - 2 \) is a factor of only the denominator, so the vertical asymptote is \( x = 2 \).

The degree of the numerator is equal to the degree of the denominator, so the horizontal asymptote is \( y = \frac{4}{1} = 4 \).

LESSON 8.3

Key Skills

Multiply, divide, and simplify rational expressions, including complex fractions.

Simplify \( \frac{x^2 + 2x - 3}{x^2 + 5x + 6} + \frac{4x^2 - 4x}{x^2 + 3x + 2} \).

\[
\begin{align*}
&= \frac{x^2 + 2x - 3 + 4x^2 - 4x}{x^2 + 5x + 6} + \frac{4x^2 - 4x}{x^2 + 3x + 2} \\
&= \frac{(x + 3)(x - 1) + (x + 1)(x + 2)}{(x + 2)(x + 3)} \\
&= \frac{x + 1}{4x}
\end{align*}
\]

Simplify complex fractions.

\[
\frac{x^2 - 1}{2x^2 - x - 15} = \frac{x^2 - 1}{4x} + \frac{4x + 4}{x^2 - 3x} = \frac{(x + 1)(x - 1)}{(x - 3)(2x + 5)} + \frac{x(x - 3)}{4(2x + 5)}
\]

\[= \frac{x(x - 1)}{4(2x + 5)}, \text{ or } \frac{x^2 - x}{8x + 20}\]

Exercises

Identify all excluded values, asymptotes, and holes in the graph of each rational function.

5. \( R(x) = \frac{2x - 3}{x^2 - 8x + 12} \)

6. \( g(x) = \frac{3x - 5}{x^2 - 25} \)

7. \( f(x) = \frac{x^2 - x - 42}{x^3 + 5x - 14} \)

8. \( r(a) = \frac{a^2 + 4a - 12}{3a^2 - 12} \)

9. \( s(x) = \frac{x^2 - 9}{3x + 5} \)

10. \( M(x) = \frac{x^4 - 10x + 9}{3x^2 - 27} \)

11. \( h(y) = \frac{2y}{6y^4 - 18y^3} \)

12. \( r(t) = \frac{t^2 - t - 4r + 4}{t^2 + t - 2} \)

Exercises

Simplify each expression.

13. \( \frac{x^2 + 6x}{10} \cdot \frac{4}{x^2 - 36} \)

14. \( \frac{3x^2 + 10x - 8}{3x^2 - 17x + 10} \cdot \frac{2x^2 + 9x - 5}{x^2 + 3x - 4} \)

15. \( \frac{4a + 8}{5a - 20} + \frac{a^2 + 3a - 10}{a^2 - 4a} \)

16. \( \frac{x^2 - 9}{6} + \frac{4x - 12}{x} \)

17. \( \frac{z}{z + 1} - \frac{z + 2}{z} \)

18. \( \frac{a + 1}{a^2} \cdot \frac{a^2}{(a - 1)^2} \cdot \frac{a}{a} \)

19. \( \frac{x + 1}{x} \cdot \frac{1}{(x + 1)^2} \)

20. \( \frac{4x^2}{6x - 3} \cdot \frac{15x}{2x - 1} \)
LESSON 8.4

Key Skills

Add and subtract rational expressions.

Simplify \( \frac{2a}{a-5} - \frac{5a}{3a+2} \).

\[
\begin{align*}
\frac{2a}{a-5} - \frac{5a}{3a+2} &= \frac{2a}{a-5} \left( \frac{3a+2}{3a+2} \right) - \frac{5a}{3a+2} \left( \frac{a-5}{a-5} \right) \\
&= \frac{2a(3a+2) - 5a(a-5)}{(a-5)(3a+2)} \\
&= \frac{a^2 + 29a}{3a^2 - 13a - 10}
\end{align*}
\]

LESSON 8.5

Key Skills

Solve rational equations.

Solve \( \frac{1}{x^2} = x \).

\[
\begin{align*}
x^2 &= x \\
1 &= x^3 \\
x^3 - 1 &= 0 \\
(x - 1)(x^2 + x + 1) &= 0
\end{align*}
\]

The only real solution is \( x = 1 \). Therefore, the only point of intersection for the graphs of \( y = \frac{1}{x^2} \) and \( y = x \) occurs when \( x = 1 \).

Solve rational inequalities.

Solve \( \frac{x}{1+x} \leq 2 \).

\[
\begin{align*}
\frac{x}{1+x} &\leq 2, \quad 1+x > 0 \quad \text{or} \quad \frac{x}{1+x} \leq 2, \quad 1+x < 0 \\
\frac{x}{1+x} &\leq 2 \\
x &\leq 2(1+x) \\
x &\leq 2+2x \\
x &\leq 2 \\
x &\geq 2 \\
x &\geq -2 \\
\end{align*}
\]

If \( 1+x > 0 \), then \( x > -1 \). Thus, \( x > -1 \) and \( x \geq -2 \), or simply \( x > -1 \).

The solution is \( x > -1 \) or \( x \leq -2 \).

The graphs of \( y = \frac{x}{1+x} \) and \( y = 2 \) verify this solution.

Exercises

Simplify each expression.

21. \( \frac{3y-5}{2y-6} + \frac{4y-2}{5y-15} \)  
22. \( \frac{9y+3}{y^2-11y+18} + \frac{y+3}{y-9} \)  
23. \( \frac{2x-3}{x^2-x-3} - \frac{3x+1}{x-3} \)  
24. \( \frac{3b-39}{b^2-7b+10} - \frac{3}{b-2} \)  
25. \( \frac{2}{x} + \frac{5}{x} \)  
26. \( \frac{2}{3} + \frac{5}{3} \)

Solve each equation.

27. \( \frac{1}{x^2+1} = \frac{1}{2} \)  
28. \( \frac{4}{x^2+1} = 1 \)  
29. \( \frac{3x-1}{x^2+2x} = -1 \)  
30. \( \frac{2}{1-x^2} = \frac{x^2}{x^2+1} \)  
31. \( \frac{1}{1-x^2} = -1 \)  
32. \( \frac{1}{x} = \frac{x+2}{x+1} \)

Solve each inequality by using algebra.

33. \( \frac{1}{x} < 1 \)  
34. \( \frac{1}{x} \geq 2 \)  
35. \( \frac{1}{x^2+1} < \frac{1}{2} \)  
36. \( \frac{1}{x^2+1} \geq \frac{1}{3} \)  
37. \( \frac{1+x}{2x+3} < 1 \)  
38. \( \frac{1+2x}{2x-1} < 2 \)

Solve each inequality by graphing.

39. \( \frac{1}{x} \geq x \)  
40. \( \frac{1}{x} < 2x \)  
41. \( \frac{x^2+x+1}{x^2+3x+2} \geq x \)  
42. \( \frac{x^3+2}{x^2+2x+1} \leq 3x \)  
43. \( \frac{1}{x^2+2x+1} > 2 \)  
44. \( \frac{1}{x^2-x+2} < x \)
LESSON 8.6

Key Skills

Find the inverse of a quadratic function.
Find the inverse of \( y = x^2 - 7x + 10 \). Interchange \( x \) and \( y \), and solve for \( y \) by applying the quadratic formula.

\[
x = y^2 - 7y + 10
\]

\[
y = \frac{-(7) \pm \sqrt{(-7)^2 - 4(1)(10-x)}}{2(1)}
\]

\[
y = \frac{7 \pm \sqrt{9 + 4x}}{2}
\]

Describe the transformations applied to the square-root parent function, \( f(x) = \sqrt{x} \).

Describe the transformations applied to \( f(x) = \sqrt{x} \) to obtain \( y = 2\sqrt{3x} - 3 + 4 \).

\[
g(x) = 2\sqrt{3x} - 3 + 4 = 2\sqrt{3(x-1)} + 4
\]

The parent function is stretched vertically by a factor of 2, compressed horizontally by a factor of \( \frac{1}{3} \), translated horizontally 1 unit to the right, and translated vertically 4 units up.

LESSON 8.7

Key Skills

Simplify expressions involving radicals.

Simplify \( \left(\frac{24a^5b^3}{3ab^2}\right)^{\frac{1}{3}} \cdot \frac{\sqrt[3]{4a^2b}}{\sqrt[3]{3ab^2}} \).

\[
\left(\frac{24a^5b^3}{3ab^2}\right)^{\frac{1}{3}} \cdot \frac{\sqrt[3]{4a^2b}}{\sqrt[3]{3ab^2}} = \frac{\sqrt[3]{24a^5b^3} \cdot \sqrt[3]{4a^2b}}{\sqrt[3]{3ab^2}}
\]

\[
= \frac{\sqrt[3]{96a^2b^3}}{\sqrt[3]{3ab^2}}
\]

\[
= \sqrt[3]{32a^{10}b^5}
\]

\[
= \sqrt[3]{2^4a^2b^4 \cdot 2a^2b}
\]

\[
= 2a^2|b|\sqrt[3]{2a^2b}
\]

Rationalize the denominators of expressions.

Write \( \frac{1}{2 + \sqrt{2}} \) with a rational denominator.

\[
\frac{1}{2 + \sqrt{2}} = \frac{1}{2 + \sqrt{2}} \left(\frac{2 - \sqrt{2}}{2 - \sqrt{2}}\right) = \frac{2 - \sqrt{2}}{2}
\]

Exercises

Find the inverse of each quadratic function.

45. \( y = 3x + x^2 \)

46. \( y = 8x + 12 + x^2 \)

47. \( y = 3x^2 - 16x + 5 \)

48. \( y = 2x^2 + 7x + 6 \)

For each function, describe the transformations applied to \( f(x) = \sqrt{x} \).

49. \( g(x) = \frac{1}{3}\sqrt{x} \)

50. \( h(x) = 3\sqrt{x} - 5 \)

51. \( k(x) = \sqrt{2x - 3} \)

52. \( g(x) = 4\sqrt{2x + 1} + 2 \)

53. \( h(x) = -2\sqrt{3x} - 6 \)

54. \( r(x) = 5\sqrt{3(x - 1)} + 1 \)

Evaluate each expression.

55. \( 5\left(\sqrt{27}\right)^2 \)

56. \( \frac{1}{2} \left(\sqrt{8} + 1 \right) \)

Exercises

Simplify each radical expression. Assume that the value of each variable is positive.

57. \( \sqrt{6x^2y^4} \cdot (3x^2y)^{\frac{1}{2}} \)

58. \( (5a^2b^3)^{\frac{1}{2}} \cdot \sqrt[3]{4a^3b} \)

59. \( \frac{\sqrt[3]{42c^2d^4}}{(6cd^{11})^{\frac{1}{3}}} \)

60. \( \frac{(45s^6t^6)^{\frac{1}{2}}}{\sqrt[3]{3t^2}} \)

61. \( \frac{(6x^5y^{\frac{1}{2}})^{\frac{1}{3}} \cdot \sqrt[3]{3x^2y^4}}{2x} \)

62. \( \frac{(24m^2n)^{\frac{1}{3}} \cdot \sqrt[3]{9m^2n^2}}{\sqrt[3]{3mn^2}} \)

Write each expression with a rational denominator and in simplest form.

63. \( \frac{1}{\sqrt{5}} \)

64. \( \frac{1}{\sqrt{7}} \)

65. \( \frac{3}{2 - \sqrt{3}} \)

66. \( \frac{4}{-2 + \sqrt{5}} \)

67. \( \frac{1 + \sqrt{2}}{3 - \sqrt{3}} \)

68. \( \frac{2 - \sqrt{3}}{3 + \sqrt{2}} \)
LESSON 8.8

Key Skills

Solve radical equations.

Solve $2x = \sqrt{3 - x}$.

\[
2x = \sqrt{3 - x} \\
4x^2 = 3 - x \\
4x^2 + x - 3 = 0 \\
(4x - 3)(x + 1) = 0 \\
x = \frac{3}{4} \quad \text{or} \quad x = -1
\]

Check for extraneous solutions.

\[
2x = \sqrt{3 - x} \\
2 \left( \frac{3}{4} \right) \leq \sqrt{3 - \left( \frac{3}{4} \right)} \\
\frac{3}{2} \leq \frac{3}{2} \quad \text{True} \\
-2 = 2 \quad \text{False}
\]

Solve radical inequalities.

To solve $\sqrt{2x - 1} \leq 1$, first solve $2x - 1 \geq 0$.

\[
2x - 1 \geq 0 \\
x \geq \frac{1}{2}
\]

Then solve the original inequality.

\[
\sqrt{2x - 1} \leq 1 \\
(\sqrt{2x - 1})^2 \leq 1^2 \\
2x - 1 \leq 1 \\
2x \leq 2 \\
x \leq 1
\]

Thus, $x \geq \frac{1}{2}$ and $x \leq 1$, or $\frac{1}{2} \leq x \leq 1$. The solution can be verified by graphing.

Exercises

Solve each radical equation by using algebra. If the inequality has no real solution, write no solution. Check your solution.

69. $\sqrt{x + 2} = -2$  
70. $3 \sqrt{x - 1} + 8 = 6$
71. $\sqrt[3]{x + 2} = -2$  
72. $3 \sqrt[3]{x - 1} + 8 = 6$
73. $\sqrt{x} = 2x$  
74. $\sqrt{x + 2} = 3$
75. $\sqrt{x} = \sqrt{-x + 3}$  
76. $\sqrt{2x - 1} = \sqrt{4x - 4}$
77. $\sqrt[3]{4 - x} = \sqrt[3]{3x}$  
78. $\sqrt[5]{2x} = \sqrt[5]{x + 3}$
79. $\sqrt{x} - 2 = \sqrt{x - 2}$  
80. $\sqrt{3x - 1} = \sqrt{x + 2}$

Solve each radical inequality by using algebra. Check your solution.

81. $\sqrt{x} \leq 5$  
82. $\sqrt{x - 1} < 2$
83. $\sqrt{x} \geq 5$  
84. $\sqrt{x - 1} > 2$
85. $\sqrt[4]{x - 2} \geq 1$  
86. $\sqrt[4]{x - 1} < 1$
87. $\sqrt{2x + 2} > 4$  
88. $-2\sqrt{x - 2} < -1$
89. $\sqrt{6x} < 0$  
90. $4\sqrt{5x - 1} < 0$

Solve each radical inequality by graphing.

91. $\sqrt[3]{x - 2} \leq \sqrt[3]{x}$  
92. $\sqrt[5]{2x + 1} \geq 2$

Applications

PHYSICS  The weight of an object varies inversely as the square of the distance from the object to the center of Earth, whose radius is approximately 4000 miles.

93. If an astronaut weighs 175 pounds on Earth, what will the astronaut weigh at a point 60 miles above Earth’s surface?

94. If an astronaut weighs 145 pounds at a point 80 miles above the Earth’s surface, how much does the astronaut weigh on Earth?
1. \( y \) varies jointly as \( x \) and \( z \). If \( y = -63 \) when \( x = 7 \) and \( z = -9 \), find \( y \) when \( x = -15 \) and \( z = -\frac{1}{2} \).

2. **CONSTRUCTION**  The strength of a beam varies directly with the width of a beam and inversely as the cube of the depth. If a beam 10 mm wide by 20 mm deep will support 1200 kg, how much will a beam 8 mm by 25 mm support?

Identify all excluded values, asymptotes, and holes in the graph of each rational function.

3. \( f(x) = \frac{x - 4}{x^2 - 16} \)

4. \( h(x) = \frac{x^2 + 2x - 15}{2x^2 - 18} \)

5. \( g(x) = \frac{2x^3 - 16}{x^3 - 2x^2 - 9x + 18} \)

Simplify each expression.

6. \( \frac{x^2 - 9}{2x^2 - 8x + 6} \cdot \frac{4x^3 - 12x + 36}{x^3 + 27} \)

7. \( \frac{3x^2 - 12}{x^3 + 5x^2} \cdot \frac{x^3 + 9x - 30}{3x^3 + 9x} \)

8. \( \frac{3x}{x - 2} \div \frac{6x^2}{2x^2 - 8} \cdot \frac{5x + 1}{2x + 4} \)

Simplify each expression.

9. \( \frac{4}{x^2 - 4} + \frac{x + 3}{x + 2} \)

10. \( \frac{x - 37}{x^2 - 2x - 15} - \frac{5}{x + 3} \)

11. **GEOMETRY**  Find the area of the shaded region of the figure at right if the largest triangle has an area \( A = x \).

Solve each equation or inequality.

12. \( \frac{x + 3}{x - 1} = 2 \)

13. \( \frac{z - 4}{z + 2} + \frac{z - 5}{z - 4} = 1 \)

14. \( \frac{3}{x + 4} \leq \frac{5}{x + 7} \)

For each function, describe the transformations applied to \( f(x) = \sqrt{x} \).

16. \( g(x) = \sqrt{x - 4} \)

17. \( h(x) = -2\sqrt{x} + 3 \)

Find the inverse of each quadratic function.

18. \( y = x^2 + x \)

19. \( y = 5x^2 - 3x - 4 \)

Evaluate each expression.

20. \( (3\sqrt{81})^2 - 31 \)

21. \( \frac{1}{3}((\sqrt{9})^3 + (\sqrt{64})^2 + 2) \)

Simplify each expression. Assume that the value of each variable is positive.

22. \( 5\sqrt{8x^3} \cdot (2x^5y)^{\frac{1}{2}} \)

23. \( \frac{8\sqrt{5r^3s^6}}{\sqrt{25r^3s^6t}} \)

24. \( (5 - \sqrt{12}) - (2\sqrt{27} + 8) \)

25. \( (2 + \sqrt{5})(3 - 2\sqrt{5}) \)

Write each expression with a rational denominator and in simplest form.

26. \( \frac{4}{\sqrt{11}} \)

27. \( \frac{1}{2 + \sqrt{5}} \)

28. \( \frac{2 - \sqrt{3}}{5 + \sqrt{7}} \)

Solve each radical equation or inequality. If no solution, write no solution.

29. \( \sqrt{2x + 7} = -3 \)

30. \( \sqrt{3x} = \sqrt{4x - 7} \)

31. \( \sqrt{x - 7} < 5 \)

32. \( \sqrt{2x + 1} \geq 3 \)
27. domain of all real numbers except \( x = -2 \) and \( x = 2 \); vertical asymptotes: \( x = -2 \) and \( x = 2 \); horizontal asymptote: \( y = 0 \); no holes

\[
\begin{align*}
\text{vertical asymptotes:} & \quad x = -2, 2 \\
\text{horizontal asymptote:} & \quad y = 0
\end{align*}
\]

29. domain of all real numbers; no vertical asymptotes; horizontal asymptote: \( y = 0 \); no holes

\[
\begin{align*}
\text{horizontal asymptote:} & \quad y = 0
\end{align*}
\]

31. domain of all real numbers except \( x = 1 \) and \( x = 6 \); vertical asymptotes: \( x = 1 \) and \( x = 6 \); no horizontal asymptote; no holes

\[
\begin{align*}
\text{vertical asymptotes:} & \quad x = 1, 6 \\
\text{no horizontal asymptote; no holes}
\end{align*}
\]

33. domain of all real numbers except \( x = 0 \) and \( x = 4 \); vertical asymptote: \( x = 4 \); horizontal asymptote: \( y = 0 \); hole when \( x = 0 \)

\[
\begin{align*}
\text{vertical asymptote:} & \quad x = 4 \\
\text{horizontal asymptote:} & \quad y = 0 \\
\text{hole when} & \quad x = 0
\end{align*}
\]

35. \( f(x) = \frac{3x}{x - 2} \)

37. \( f(x) = \frac{3x^3}{2x^2 - 2x} \)

39. \( f(x) = \frac{x^3 - 2x^2}{x^2 - 2x} \)

41a. \( c > \frac{9}{2} \) if there are no vertical asymptotes, then \( x^2 - 3x + c = 0 \) has no solutions and \( b^2 - 4ac < 0 < 9 - 4c < 0 \). \( b. \) \( c = \frac{9}{2} \) if there is 1 vertical asymptote, then \( x^2 - 3x + c = 0 \) has 1 solution and \( b^2 - 4ac = 0 \), \( 9 - 4c = 0 \). \( c. \) \( c < \frac{9}{2} \) if there are 2 vertical asymptotes, then \( x^2 - 3x + c = 0 \) has 2 solutions and \( b^2 - 4ac > 0 \), \( 9 - 4c > 0 \). \( 43a. \) \( R(x) = \frac{2}{x} \)

47. \( C(x) = \frac{11.45x + 250}{x} \)

49. \(-\frac{4}{5} \leq x < 2 \quad 51. \) \(-36x^2 + 24x \)

53. \( 3x^2 - 2x + 2 \)

55. \( 9x^2 - 16 \)

57. \( (x - 1)(x^2 + x + 1) \)

61. \( (x + 6)^2 \)

41b. \( c < \frac{9}{2} \) if there are no vertical asymptotes, then \( x^2 - 3x + c = 0 \) has no solutions and \( b^2 - 4ac < 0 < 9 - 4c < 0 \). \( b. \) \( c = \frac{9}{2} \) if there is 1 vertical asymptote, then \( x^2 - 3x + c = 0 \) has 1 solution and \( b^2 - 4ac = 0 \), \( 9 - 4c = 0 \). \( c. \) \( c > \frac{9}{2} \) if there are 2 vertical asymptotes, then \( x^2 - 3x + c = 0 \) has 2 solutions and \( b^2 - 4ac > 0 \), \( 9 - 4c > 0 \). \( 43b. \) \( a, b \) all real numbers greater than 0; all real numbers greater than 0; all real numbers greater than 0

45a. \( V = \ell \times w \times h \); \( V = (20 - 2x)(16 - 2x) \times \frac{2x(20 - 2x) + 2x(20 - 2x) + 2x(16 - 2x) - 320 - 40x - 32x + 4x^2 + 40x - 4x^2 + 32x - 4x^2}{80 - x^2} \)

47a. \( d = \frac{d_{1} - d_{2}}{t_1 - t_2} \)

51. \( y = -5x + 20 \)
LES SSON 8.4

TRY THIS (p. 505)
a. \( \frac{5x + 4}{2x - 1} \) b. 2

TRY THIS (p. 506)
\[ x^2 - 5x - 5 = \frac{2x^2 - 5x - 5}{x + 5(x - 5)} \]
\[ x^2 - 25 \]

TRY THIS (p. 507, Ex. 3)
\[ \frac{5x + 12}{x(x - 2)(x + 2)} = \frac{507 + 12}{x^2 - 4x} \]

TRY THIS (p. 507, Ex. 4)
\[ \frac{2a^2}{(a + 1)(a - 1)(a^2 + 1)} = \frac{2a^2}{a^2 - 1} \]

Exercises
4. \( \frac{3x + 5}{x - 1} \) 5. 2 6. \( \frac{4x + 8}{x + 1(x - 1)} = \frac{4x + 8}{x^2 - 1} \)
7. \( \frac{-3x^2}{(2x - 1)(x - 1)} = \frac{-3x^2}{2x^2 - 3x + 1} \)
8. \( \frac{43.9}{t - 1} \)
9. \( \frac{4x - 16}{9} = \frac{4x - 16}{x^2 - 4} \)
10. \( \frac{37x - 11}{(3x - 5)(2x + 3)} = \frac{37x - 11}{6x^2 - x - 15} \)
11. \( \frac{3x^3 + 3x - 2}{(2x - 1)(x - 1)} = \frac{3x^3 + 3x - 2}{2x^2 - 3x + 1} \)
12. \( \frac{x^2 - 2x + 3}{(x - 1)^2} = \frac{x^2 - 2x + 3}{x^2 - 2x + 1} \)
13. \( \frac{3x}{2x - 1} = \frac{2x + 1}{x - 1} \)
14. \( \frac{x^2 + x + 16}{(x - 3)(x + 4)} = \frac{x^2 + x + 16}{x^2 + x - 12} \)
15. \( \frac{4ab}{(a - b)^2} = \frac{4ab}{a^2 - 2ab + b^2} \)
16. \( \frac{2r^2 + 5rs + 2s^2}{(2r - s)(2r + s)} = \frac{2r^2 + 5rs + 2s^2}{4r^2 - s^2} \)
17. \( B = 1 \) and \( D = -2 \)
18. \( A = \frac{1}{6}, C = \frac{2}{3} \) and \( D = -\frac{7}{3} \)
19. \( = 48.8 \) mph

b. \( R_T = \frac{R_1R_2R_3}{R_1R_2 + R_2R_3 + R_3R_1} \)
20. \( = 48.8 \) mph

Distribution Property 53. \( -7 \) 54. \( x = \frac{3}{2} \pm \frac{\sqrt{2}}{2} \)
55. \( 121; 2; x = \frac{3}{2} \) or \( x = -4 \)
56. \( \log_2 \frac{32}{8} \) or \( \log_3 4; 2 \)
57. \( x = 2 \) or \( x = -1 \) or \( x = -3 \)

LESSON 8.5

TRY THIS (p. 513)
\( x = -1 \) or \( x = 3 \)

TRY THIS (p. 514)
\( x = \frac{1}{2} \)

TRY THIS (p. 515)
\( x > -2 \) or \( x < -\frac{3}{2} \)

TRY THIS (p. 516)
\( x < 1 \)

Les sson 8.6

TRY THIS (p. 521)
\( x \geq \frac{-18}{5} \) or \( x \geq -3.6 \)

TRY THIS (p. 522)
a. \( \) a vertical stretch by a factor of 3 and then a vertical translation 2 units down, and a horizontal translation 1 unit to the right
b. \( \) a horizontal compression by a factor of \( \frac{1}{2} \), a vertical translation 3 units up, and a horizontal translation \( \frac{1}{2} \) unit to the left

TRY THIS (p. 523)
Interchange the roles of \( x \) and \( y \).
\( y = x^2 + 3x - 4 \rightarrow y = x^2 + 3y - 4 \)
\( y = -3 \pm \sqrt{25 + 4x} \)

TRY THIS (p. 524)
\( 0, 26 \)
51. \(-1.4 \leq x \leq -1\) or \(1 \leq x \leq -1.6\)  
53. never  
55. never  
57. sometimes  
59. always  
61a. \(a < 0\)  
63a. \(191,916\) feet  
65. \(6.3\) miles  
67. \(x = -2\) or \(x = -3\)  
69. \(x = 0.8\) or \(x = -2.1\)  
71. \(x^2 - 6x - 27\)  
73. \(24x^3y^3\sqrt{2x}\)  
75. \(x\sqrt{5}\)  
77. \(2x\sqrt{10}\)

### Chapter 9

#### LESSON 9.1

TRY THIS (p. 563)  
a. ellipse \((y = \pm\frac{1}{3}\sqrt{40} - 4x^2)\)  
b. parabola \((y = \pm\sqrt{6}x)\)  

TRY THIS (p. 564)  
\(\sqrt{194}\), or about 13.93